IPM – Component 3

Critiquing (with a comparison to summarising and paraphrasing)

In order to be able to critique, as well as summarise and paraphrase, a topic you have to be sufficiently familiar with the topic. Since the only topic I am familiar with in sufficient depth is mathematics I will present examples relating to mathematics and statistics.

Example 1: Statistics – On means, medians and modes in statistics

Consider the following text (which I have invented):

> Here we describe the mean and median as ways of finding averages in data. The mean is a way of calculating the average value of data using simple arithmetic, whereas the median does not rely on arithmetic. Instead, the median relies of ordering the data from smallest to largest and then identifying the middle value. This middle value is a form of average because it represents the most common value among the data.
>
> Focusing more closely on the nature of data itself, we find that there are data values called outliers. Outliers can be defined as data values which lie significantly outside the main trend of the data. A single outlier in a data set can significantly change the value of the mean compared to the mean calculated without the outlier. On the other hand, the median is not affected by outliers. When data is arranged in order the middle data values remains in the middle, irrespective of how large or small the outlier is.

Now consider the following summary and paraphrase to these two texts:

1. *Summary*: The text above mentions two ways in which we can find the average of a data set, namely the mean and the median. It also describes how outliers affect the result of the mean and median.
2. *Paraphrase*: Two standard measures of central tendencies are the mean and median. The mean is found by summing the data and then dividing by the number of data. In other words, it is the usually understood way of finding an average. On the other hand, the median relies on the data being put into ascending order, from which we locate the

middle value. Because the value is in the middle of the ordered data set it is considered the most common value.

In terms of the data values themselves we find that there are such things as outliers. Outliers are values which are considered unusual because instead of following the main trend of the data they lie at an abnormal distance from all other values. They are considered extreme values since they stand out greatly from the overall pattern of the data set. Outliers can significantly affect the value of the mean but not the value of the median since the ordering of data is not affected by the size of outlying values.

Questions

1. What is it that makes the summary a summary? What is it that makes the paraphrase a paraphrase?
2. Is it possible to give more detail to the summary above, and it still be a summary? If so, to what extent can we include more detail?
3. Is it possible to shorten the paraphrase above, and it still be a paraphrase? If so, to what extent can we make the paraphrase less detailed?

Example 2: Mathematics (ODEs in modelling)

The following text is adapted from "Rethinking pedagogy for second-order differential equations: a simplified approach to understanding well-posed problems", Christopher C. Tisdell, *International Journal of Mathematical Education in Science and Technology*, 2017, 48:5, 794-801.

> "An important question arising from the modelling and analysis of differential equations involves asking whether or not the given differential equation subject to some initial conditions is well posed. That is, we want to know if the problem has exactly one solution and to have simple sufficient conditions at hand which we can apply to a wide range of cases to verify the 'well posedness' of a given problem. If the problem has no solution, or multiple solutions, then it is not well posed from a modelling point of view and it must be discarded and a new model formulated.

Now consider the following two description of this text:

1. *Summary*: The author discusses the well-posedness of differential equations with respect to how the teaching of this can be made more direct.
2. *Paraphrase*: The well-posedness of second- or higher order differential equations (DEs) has usually been addressed by reducing the DEs to a system of first order DEs. This is done in order to determine whether or not the original DE has exactly one solution. From a modelling perspective, should the DE not have a unique solution then a new model needs to be created.

   From the perspective of teaching and learning about well-posedness this reduction of second- or higher-order DEs to a system of first order DEs makes for more work than necessary. A more direct approach to answering the question of well-posedness is presented here.

Questions

1. What is it that makes the summary a summary? What is it that makes the paraphrase a paraphrase?
2. Is it possible to give more detail to the summary above, and it still be a summary? If so, to what extent can we include more detail?
3. Is it possible to shorten the paraphrase above, and it still be a paraphrase? If so, to what extent can we make the paraphrase less detailed?

Commentary about summaries and paraphrases

We can summarise texts we ourselves have written (such as in the summary of our own essay) or texts written by someone else. Ditto for paraphrasing. With this in mind I might define the term *primary writing* as wrting which refers to our own text, and *secondary writing* as writing which refers to other people's texts (note that the italicised terms are my personal wording and are not conventional when talking about genres/style of writing). So that

- summaries can be considered as primary or secondary writing: we can summarise our own work or other peoples' work;
- paraphrases can also be considered as primary and secondary writing: we can certainly paraphrases other people's work but we can also paraphrase our own work

in order to give a simpler, less technical description of something we have previously written.

So how do we tell the difference between primary and secondary type writing? By the use of reference-type language (not to be confused with references as a list of books and papers included at the end of an essay). The following are examples of phrasing used as reference-type language one can use when summarising:

1a) "Prior work by … reveals that …": this references other peoples' work;

2a) "Earlier in this paper I stated that …": this references something I said earlier in the paper;

3a) "It is generally accepted that …"; this references an idea, theory, opinion, etc. that the community of experts agree upon;

4a) "In summary we have …": this references something previously stated/written, either in the same paper or from another paper;

5a) "Author X highlights similarities and difference between …": this references a specific person's critique;

6a) "However, as discussed in Smith (2000) …": this references the comments made by a specific author;

etc. Sentences involving the type of phrasing above can then be seen to be generalistic in style, i.e. they do not present any detail about the topic mentioned:

1b) "Prior work by *Smith (2000) on the mean of random samples taken from a single population* reveals that *these means have a natural variation*". Two possible aspects of detail missing here could relate to

i) the distribution of the population: is it normally distributed? Is it skewed? Etc.;
ii) the size of the samples taken: too small a sample size adversely affects the sample means. There is a greater variety in the means for small sample sizes compared to large sample sizes;

Hence 1b) is classed as a summary because of the phrases used and the generality of the description;

3b) "It is generally accepted that *the mean is susceptible to outliers whereas the median is not*". Two possible aspects of detail missing here could relate to

i) how extreme the outliers are: the more extreme the outlier, the greater its effect on the mean;

ii) how the median is unaffected by extreme values: since data is arranged in ascending order the size of the outliers has no effect on the middle value of that ordered data.

Hence 3b) is classed as a summary because of the phrases used and the generality of the description;

The table below illustrates more examples of types of phrasing and sentence development. The aim of this table is to show you examples on an *underlying principle* of what constitutes *summary* language and description. This underlying principle is what you should aim to learn and understand. Then you will know *how* to paraphrase or *how* to write a summary, and you will only need to learn individual vocabulary, terminology, and phrasing in order to paraphrase or summaries in *your own words*.

{prior / previous / past / …}	{studies / work / research / data / …}	by author A on …	{reveals / indicates / shows / illustrates / …}	{important / fundamental / crucial / significant / …}	{similarites / differences / changes / deviatiions / …}	in … / with respect to …
Author A	{describes / highlights / criticises / defines / …}	{similarites / differences / changes / deviations / …}	in … / with respect to …	{as a result of / due to / because of / by reason of / in order to account for / …}	…	

<u>Two examples of suitable language for writing summaries</u>

When one is paraphrasing one generally tends to write at the same level of detail and depth (more or less) as the original text, but this time using one's own words. This means that we use alternative phrasing, and we construct sentences or paragraphs differently to that of the original text, whilst keeping the same emphasis and meaning of the original text.

For example, consider the following text as a primary piece of writing:

> In his paper we discuss the mean as a measure of central tendency, explaining that the advantage of this is that, because all the data values are used in finding the mean, taking the mean for different samples of a population tends to give similar results. This indicates that the mean is robust, namely that it resists very well any fluctuations between different samples.

we can paraphrase this as illustrated below whilst retaining the essential meaning of the text:

- The advantage of using the mean as a measure of central tendency are discussed. Examples of finding means for different samples from the same population are presented, the results of which indicate that these means are similar. Such an outcome illustrates that there is little fluctuation between different samples. Because of this the mean is said to be robust.

or

- This paper describes, with examples, how it is that means calculated from different sample of the same population is robust. Robustness can be defined as the ability of data to resit fluctuation to a certain degree. This proves to be one of the great advantages of this measure of central tendency.

Compare the above paraphrases with the following summaries:

- The particular type of average known as *mean* is discussed with reference to the advantages it brings when used in analysis data.

or

- A particular quality of the mean data, known as *robustness*, is presented along with its benefits.

Exercises

1) Do the summaries above keep the essential meaning of the original text?
2) Do the summaries above focus on the same theme as each other? As the original text?

So what is a summary and a paraphrase?

So what is the difference between a summary and a paraphrase? Well,

1. The aim of a summary is to report in a brief and yet accurate manner the main idea, objective, methodology, results, and success/failure of intended aim of the original paper. The goal of summary is not to offer an evaluation or opinion of the original article. A summary is far more concise than the original paper. It is a self-contained piece of writing which is fully formed and able to make sense on its own.

   As such, the summary of example 1 on p1 is short and mentions only the main or essential theme of the text.

2. The aim of a paraphrase is to demonstrate your own understanding of a text, and do so in your own words. Paraphrases are longer than summaries and approximately the same length and level of detail as the original text. This requires two things: i) an understanding of the topic of the text, ii) a certain ability at using language to write one's understanding.

   As such, the paraphrase of example 1 on p1 goes into more detail then the summary and does so in my own words rather than repeating phrases or sentences of the text. My paraphrases uses synonyms but is not simply just this. Nor, generally, does my paraphrase reorder phrases or sentences, or copy phrasal or sentence structure of the original text. I do this latter only if I am quoting *or if the phrasing is standard technical phrasing used in the discipline* (as in "singnificance testing", "confidence intervals", "performing a *t*-test at the 5% confidence level", etc.)

It is only by doing a lot of reading that you come to know what paraphrases and summaries look like, how they are written, and how to write them yourself. This leads us to the fact that a summary or a paraphrase is seen to be a summary or a paraphrase by the use of a particular type of vocabulary constructed via phrasing and sentences and paragraphs in a particular way.

Critiquing

We now move on the aspect of critiquing a text. Generally. one cannot critique a text without first summarising or paraphrasing it. Hence, in order to see critiques in context, summaries and paraphrases will also be showed in the examples below, alongside critiques. We start with an example before moving on to defining the nature of a critique. We can use this approach because we already know what summaries and paraphrases, and we can therefore compare the examples of critiques against them. We can then use such a comparison to develop an understanding of what critiques are.

Example 1: Statistics – On means, medians and modes in statistics

Consider the following text which is an extension of example 1 on p1:

> In his paper (1980) author A states that he uses the mean as his measure of central tendency, explaining that the advantage of this is that, because all his data values are used in finding the mean, taking the mean for different samples of a population tends to give similar results. This indicates that the mean is robust, i.e. it resists very well any fluctuations between different samples.
>
> However, the problem with using the mean is that it is sensitive to outliers. The further the outliers(s) the more it will affect the mean, resulting in a value of the mean which is not representative of the "middle" of majority of the data. So one single value can significantly skew the value of the mean away from the most representative average.
>
> It may be that the data used by Author A had no outliers. However, this is not addressed, nor is any other reason relating to problems in the use of the mean as a measure of central tendency.
>
> In terms of measuring central tendency, author B's paper (1990) states the use of the median. In this paper, author B reports that this has the advantage of overcoming the problem caused by outliers when using the mean. He then goes onto to say that the disadvantage of the using the median as a measure of central tendency is that, because the median is not calculated arithmetically (and therefore does not use each data value) it is easily affected by the type of sample we take from the population.

Furthermore, both papers use only one measure of central tendency when analysing their data. It seems that their research would have benefited from using both measures as well as an analysis of the difference between these two measures as used on their respective data sets. This could then have provided answers to which types of data are more suited to using the mean and which are more suited to using the median.

We can then write the following summary, paraphrase and critique:

- *Summary:* This text summarises and critiques two different authors' choice of measures of central tendency.
- *Paraphrase:* The mean, as a measure of central tendency, is known to resist very well any fluctuations between different samples. This is known as *robustness*. It is for this reason that author A used the mean in his study on populations. This choice afforded him a degree of robustness in the analysis of the data.
- *Critique:* Using the mean as an average can be problematic since the mean is sensitive to outliers. The larger the outlier the less likely the mean is to represent the most common value. However, this is not considered in the text above. The problem of outliers can be remedied by using the median, but this too has a problem, namely that of being sensitive to the sample chosen from a population.

  Furthermore, both papers use only one measure of central tendency when analysing their data. It seems that their research would have benefited from using both measures as well as an analysis of the difference between these two measures as used on their respective data sets.

Notice that the text above is already a critique. Why? How so? Compare this example with example 1 on p1.

<u>*Exercise:*</u>

Is my summary a summary? Is my paraphrase a paraphrase? Is my critique a critique?

a) If not, why not? What is wrong with my summary, paraphrase and/or critique? How would you correct these so that they read as a summary, paraphrase and critique?

b) If so, what is it that makes then summaries, paraphrases and critiques? What type of language or phrasing or intention am I using in my critique? Which of a. to d. have I used? If I haven't used any of these forms of analysis what form of analysis have I used?

Example 2: Mathematics (ODEs in modelling)

Consider the following text which is an extension of example 2 on p2:

> "An important question arising from the modelling and analysis of differential equations involves asking whether or not the given differential equation subject to some initial conditions is well posed. That is, we want to know if the problem has exactly one solution and to have simple sufficient conditions at hand which we can apply to a wide range of cases to verify the 'well posedness' of a given problem. If the problem has no solution, or multiple solutions, then it is not well posed from a modelling point of view and it must be discarded and a new model formulated.
>
> For over 70 years, learning and teaching approaches concerning the well posedness of second-order (and higher-order) initial value problems (IVPs) have involved a significant detour. Scholars have reduced second-order (and higher-order) problems to first-order systems of equations through a transformation and then performed an analysis on the resultant system. We show that this excursion is unnecessary and present a direct approach regarding second- and higher order problems."

The last paragraph of this text represents the critique. The key phrasings, and development of the paragraph, which suggests this is highlighted in the text below:

> "For over 70 years, learning and teaching approaches concerning the well posedness of second-order (and higher-order) initial value problems (IVPs) have involved a significant detour …"

This can be more generally seen to represent

doing something (learning and teaching)
on/about something (well-posedness)
has had the *impact, or effect of* (significant detour).

The text then goes on to describe what this "detour" is, namely that

> "Scholars have reduced second-order (and higher-order) problems to first-order systems of equations through a transformation and then performed an analysis on the resultant system."

In other words,

Scholars have done something (reduced problems)
in a certain way (by a transformation)
and then *taken action as a result* (performed an analysis).

The authors then suggest an alternative way which they consider better:

> "We show that this excursion is unnecessary and present a direct approach regarding second- and higher order problems."

Example 3: Statistics – On correlation coefficients

The following text is taken from "The Absolute Correlation Coefficient", Christopher Bradley, *The Mathematical Gazette*, Vol. 69, No. 447 (Mar., 1985), pp. 12-17.

> The two most common measures of central tendency and dispersion statistics are the mean and standard deviation on the one hand, and the median and absolute deviation on the other. For most purposes the former measure is preferred for two very good reasons; the first is that the squares of quantities are easier to handle analytically than their moduli; and secondly for all the common symmetrical distributions […] if a sample is taken to estimate the central value, then the mean of that sample has a smaller variance than the median, and is therefore relatively more efficient as an estimator of the central value of the parent population.
>
> The preference for the mean and standard deviation is so pronounced by the time the topic of correlation is studied that little, if anything, is ever written about a possible analogue of the product moment correlation coefficient. So […] I thought it might be interesting to show that a theory of absolute correlation can be constructed, which is based on the measure of median and absolute deviation."

We can then write the following summary, paraphrase and critique:

- *Summary:* This text describes two reason why the mean and standard deviation are preferred over the median and absolute deviation as measures of central tendency and dispersion. Then, focusing on correlation the author wishes to develop a theory of correlation coefficient based on the median as opposed to the usual one based on the mean.
- *Paraphrase:* The mean and standard deviation dominate current statistical practice for two reasons: i) the squaring effect makes mathematical analysis easier, and ii) the sample mean will be a better estimate of the population mean and the sample median. By the time we get to studying correlation coefficients the median and absolute deviation have been completely sidelined as measures of centre and dispersion, so that we are left with only the mean and standard deviation in any future statistical theory. the author wishes to correct this by presenting a version of correlation coefficient which corresponds to the median and absolute deviation.
- *Critique:* Absolute deviation uses, by definition, the absolute value function. The great problem with such a formulation lies in the fact that the absolute value function is not differentiable at $x = 0$. Any theory which relies on the derivative of the absolute deviation will therefore need to exclude that point at which the function is not differentiable. This means that it will be impossible to analyse any rate of change of correlation between two data sets at that specific point.

Exercise:

Is my summary a summary? Is my paraphrase a paraphrase? Is my critique a critique?

a) If not, why not? What is wrong with my summary, paraphrase and/or critique? How would you correct these so that they read as a summary, paraphrase and critique?

b) If so, what is it that makes then summaries, paraphrases and critiques? What type of language or phrasing or intention am I using in my critique? Which of a. to d. have I used? If I haven't used any of these forms of analysis what form of analysis have I used?

Critiquing the technicality of statistics

Here, as for mathematics, in order to critique statistics itself we need to know statistics, and we also need to know how to present our results and conclusion properly. The specific focus of the critique is on the appropriateness of the type of test used and the interpretations of the results (i.e. level of significance or the interval of confidence).

Since statistics is not my main area of maths I won't show an actual stats example for which I could produce a technical summary, paraphrase and critique. But I know enough to know what technical areas of statistics could be critiqued. Some of these are

1. *Data collection*: What type of data are we collecting? Is the data of the right type for what we want to study, analyse and come to conclusions about? Have we collected a sufficient amount of data?
2. *Sampling techniques*: What type of sample are we going to choose? Random? Stratified? Some other type? Is our sampling procedure appropriate for what we are going to study, analyse and come to conclusions about?
3. *Probability distributions*: Are we studying a discrete distribution or a continuous one?

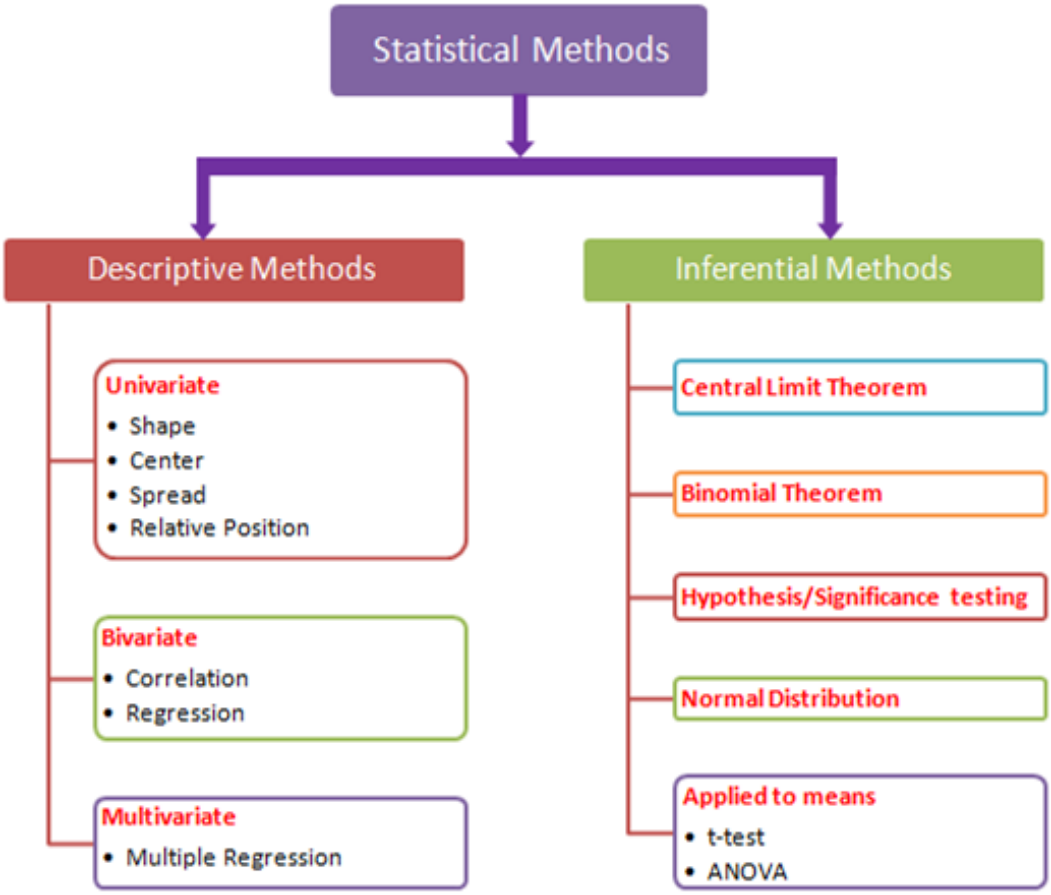
   a. *Discrete distributions*: Examples include the Binomial, Geometric or Poisson distributions. Here one would need to consider how appropriate it was to use one distribution over another.

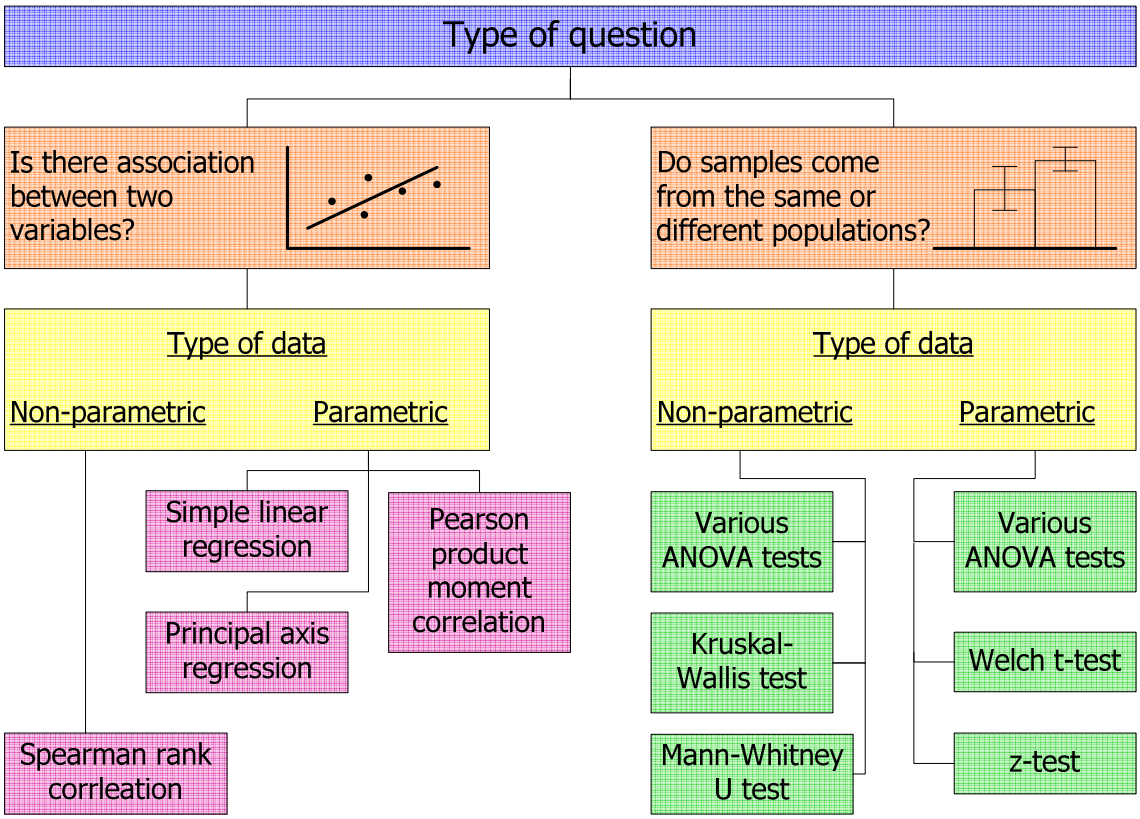
   b. *Continuous distributions*: Examples include the normal distribution, the chi-squared distribution, the gamma distribution. As for the discrete case, one would need to consider how appropriate it was to use one distribution over another.
4. *Test statistic*: What statistic of the sample are we testing? The mean? The median? The standard deviation? Was the relevant spread considered? In other words, should we have used variance instead of standard deviation?
5. *Statistical analysis*: Was the appropriate analysis conducted? In other words, was testing statistical significance appropriate? Or would it have been more appropriate to set up a confidence interval?

   If we are testing variance was the appropriate analysis of variance method used (there are several different types of methods for analysing variances).

6. *After all is said and done:* Have the results been clearly explained and appropriately interpreted? What is the significance of the results? What assumptions have been made? standard deviation? Was the relevant spread considered? In other words, should we have used variance instead of standard deviation

Other technical aspects of statistics for which a critique could be provided is illustrated in the diagrams below. One point of contention when using statistics is usually whether or not the method used was appropriate for the type, and quantity, of data collected.





Another point of contention comes from the use of something called *p*-values which is the probability we need for some effect to be significant and beyond mere chance. In fact, there is a great deal of contention in the academic community about the validity or relevance of using *p*-values (probability values which act as boundaries between significant and non-significant results), and associated significance levels. Such contention comes about as a result of, amongst other example, the many medical trials conducted to prove the efficacy of a new drug, results which other people or organisations are unable to repeat or replicate. Similar problems of repeatability and replication of statistical results using p-values are seen in the social sciences. The reason for the contention is because we are the ones who choose the *p*-value for our experiments. It is we who decide where the boundary lies between results which will be considered statistically significant or not. *P*-values tend to be set as 0.1, 0.05 or 0.01 (i.e. a 10%, 5% or 1% level of significance), and a result which is statistically significant when we set one *p*-value may not be if we set another *p*-value.

State your hypothesis

Null hypothesis: A pizza company thinks customers eat an average of 4 slices per day.

Alternative hypothesis: You think the average is higher

Collect data

For this example, data collected on number of slices eaten gives an average of 5.6 slices

Test the result

Choose your significance level: p-value = 0.05 (5%)

Choose a test: for example z-test, t-test, or other

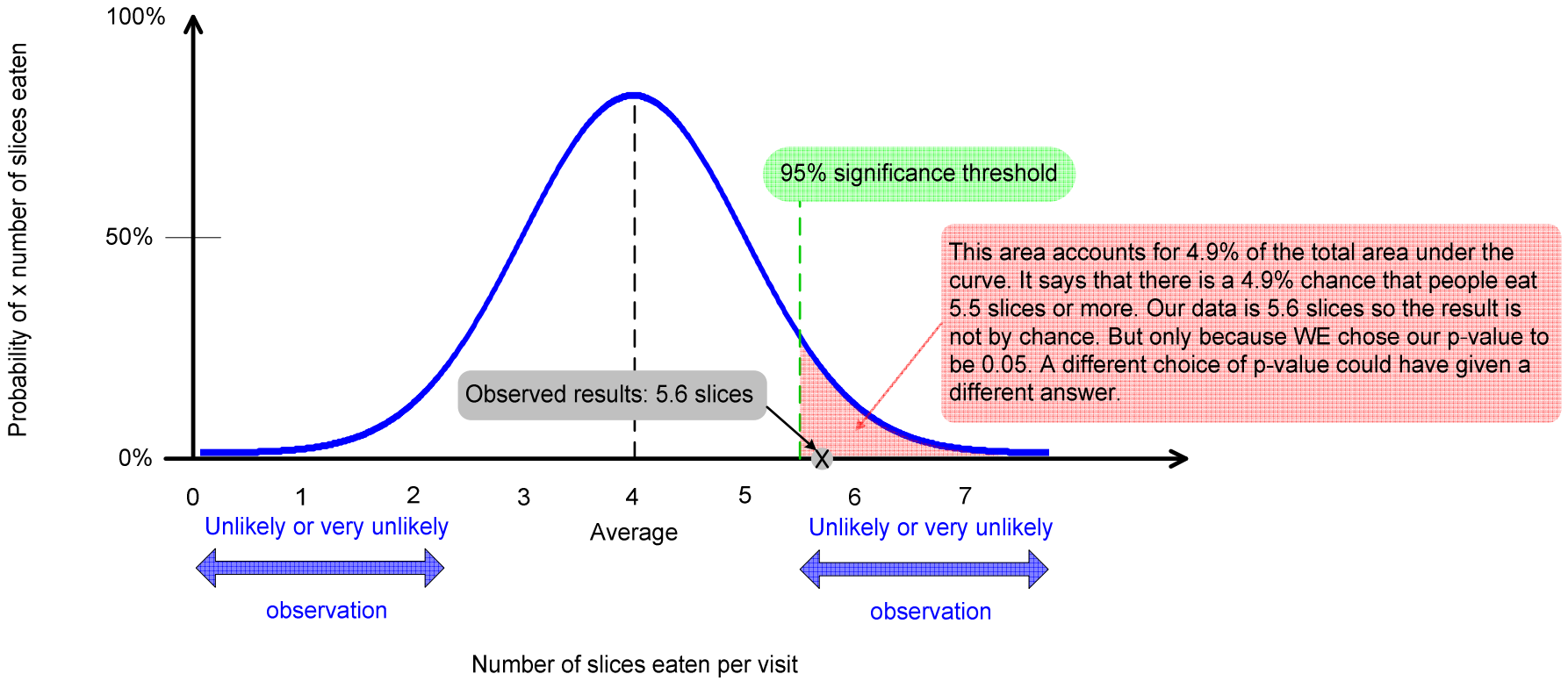
Is the result significant?

p > 0.1 : not significant

p < 0.1 : marginally significant

p < 0.05 : significant

p < 0.01 : very significant



Example of critiquing mathematics: Mathematical modelling with ODEs

Consider the text below which is adapted from "Rethinking pedagogy for second-order differential equations: a simplified approach to understanding well-posed problems", Christopher C. Tisdell, *International Journal of Mathematical Education in Science and Technology*, 2017, 48:5, 794-801. Although it refers to ordinary differential equations it is a general enough for you to understand what it is trying to say.

> "An important question arising from the modelling and analysis of differential equations involves asking whether or not the given differential equation subject to some initial conditions is well posed. That is, we want to know if the problem has exactly one solution and to have simple sufficient conditions at hand which we can apply to a wide range of cases to verify the 'well posedness' of a given problem. If the problem has no solution, or multiple solutions, then it is not well posed from a modelling point of view and it must be discarded and a new model formulated.
>
> For over 70 years, learning and teaching approaches concerning the well posedness of second-order (and higher-order) initial value problems (IVPs) have involved a significant detour. Scholars have reduced second-order (and higher-order) problems to first-order systems of equations through a transformation and then performed an analysis on the resultant system. We show that this excursion is unnecessary and present a direct approach regarding second- and higher order problems."

Consider the following summary, paraphrase and critique of the text above:

- *Summary*: The author discusses the well-posedness of differential equations with respect to how the teaching of this can be made more direct.
- *Paraphrase*: The well-posedness of second- or higher order differential equations (DEs) has usually been addressed by reducing the DEs to a system of first order DEs. This is done in order to determine whether or not the original DE has exactly one solution. From a modelling perspective, should the DE not have a unique solution then a new model needs to be created.
- *Critique:* From the perspective of teaching and learning how to solve DEs the reduction of second- or higher-order DEs to a system of first order DEs makes it much easier to solve such DEs (despite the fact that there is more work involved as a result of such

reduction). However, there is a question as to how such a reduction approach helps in the teaching and learning of the well-posedness of DEs. A more direct approach to developing an understanding of well-posedness of higher order DEs can be taken by doing ….

Exercise:

1) Can you identify the vocabulary and/or type of phrasing used when writing a summary and paraphrase?

2) My critique of example 1 is clearly a critique and not a summary or paraphrase. What is it about my critique that makes it a critique? What type of language or phrasing or intention am I using in my critique? Which of a. to d. have I used? If I haven't used any of these forms of analysis what form of analysis have I used?

Critiquing the technical aspects of mathematics

When it comes to critiquing the technical aspects of a paper you need to know the topic of the paper. Again, since mathematics is the only topic I know well enough there is no getting around the fact that I will have to present a piece of maths in order to illustrate the critique of mathematics itself. The first two examples are examples based on simple arithmetic so you should all be able to follow the maths of these. In the next section I will present more advanced examples (A-level maths examples) which you might remember doing at school.

Example 1: Various methods of multiplying two number

There are many ways of multiplying two numbers, some of which are shown below. Each has similarites and differences, its advantages and disdavantages. The different approaches to multiplication then allows us to develop a critique. We shall do this at the end of method 5.

Method 1: The usual way: Long multiplication

$$\begin{array}{rr} & 325 \\ \times & 12 \\ \hline & 650 \\ & 3250 \\ \hline & 3900 \end{array}$$

Method 2: Repeated addition

$$325 + 325 + 325 + 325 + 325 + 325 + 325 + 325 + 325 + 325 + 325 + 325 = 3900$$

Method 3: Split the multiplier into a binomial term and expand

Version a

$$\begin{aligned} 325 \times 12 &= 325 \times (10 + 2) \\ &= 325 \times 10 + 325 \times 2 \\ &= 3250 + 650 \\ &= 3900 \end{aligned}$$

Version b

$$\begin{aligned} 325 \times 12 &= 325 \times (8 + 4) \\ &= 325 \times 8 + 325 \times 4 \\ &= 2600 + 1300 \\ &= 3900 \end{aligned}$$

Method 4: Halving and doubling.

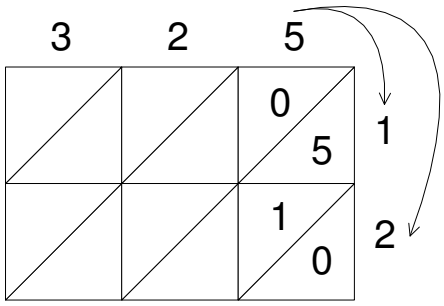
For 14 × 12, halve one number (ignoring any remainder) until you get to 1, and double the other number. Then add all doubled numbers that lie across odd halved numbers:

~~14~~	~~12~~
7	24
3	48
1	96

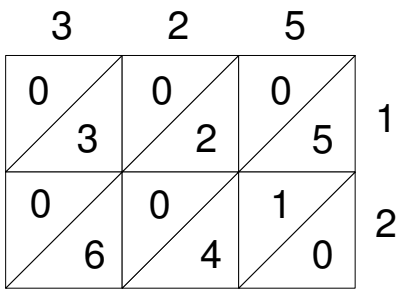
So 14 × 12 = 24 + 48 + 96 = 168

Method 5: Grid multiplication

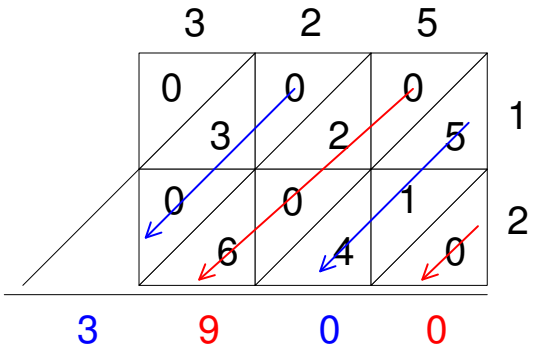
Multiply each digit with each other digit and place in the relevant triangle. For example



So we get



Finally, add along the diagonals, carrying as needed:



Now for the critique

What are the similarites and differences, advantages and disdavantages, etc. of these methods? Well …

1) Method 1 = method 5 because the lay out both numbers in hundreds, tens and units,
2) Method 1 ≠ method 5 because (in one sense) method 5 allows use to multiply the digits independently of each other, whereas method does not (although this is not strictly true!)
3) Method 4 is completely different from all the other methods since it relies only on multiplying and dividing by 2, and taking account of the rmainder of such a division.

etc.

Exercise: Below is another approach to multiplication. Compare and contrast this method with those above, and write a critique of this method.

$$97 \times 91 = \mathbf{8827}$$

$$100 - 97 \quad 100 - 91 \qquad 100 - 12 = \mathbf{88}$$

$$3 + 9 = 12$$

and

$$3 \times 9 = \mathbf{27}$$

For your own interest, the following (which does not need to form part of your critique but can do so if you wish) justifies the mathematical validity of the above process:

$$\begin{aligned} 97 \times 91 &= (100-3)(100-9) \\ &= 100^2 - 100 \times 9 - 100 \times 3 + 3 \times 9 \\ &= 100^2 - 100(3+9) + 3 \times 9 \\ &= 100[100-(3+9)] + 3 \times 9 \\ &= 100 \times 88 + 27 \\ &= 8827 \end{aligned}$$

Critiquing, summarising, and paraphrase: A comparison

As we can see from the previous example, in order to critique mathematics itself we need to know mathematics. But we also need to know how to present it formally. For pure mathematics there is only one focus of the critique, and that is on the correctness of the mathematics. For applied mathematics, where mathematics is used to model real world phenomena, the focus is also on the appropriateness of the mathematical model, the assumption used in the model, and the interpretations of the results obtained from the model.

Then we may describe the difference between a summary, a paraphrase and a critique as follows (where I have deliberately repeated the description of summary and paraphrase in order to make these more accessible when comparing with a critique):

1. The aim of a summary is to report in a brief and yet accurate manner the main idea, objective, methodology, results, and success/failure of intended aim of the original paper. The goal of summary is not to offer an evaluation or opinion of the original article. A summary is far more concise than the original paper. It is a self-contained piece of writing which is fully formed and able to make sense on its own.

   As such, the summary of example 1 on p1 is short and mentions only the main or essential theme of the text.

2. The aim of a paraphrase is to demonstrate your own understanding of a text, and do so in your own words. Paraphrases are longer than summaries and approximately the same length and level of detail as the original text. This requires two things: i) an understanding of the topic of the text, ii) a certain ability at using language to write one's understanding.

As such, the paraphrase of example 1 on p1 goes into more detail then the summary and does so in my own words rather than repeating phrases or sentences of the text. My paraphrases uses synonyms but is not simply just this. Nor, generally, does my paraphrase reorder phrases or sentences, or copy phrasal or sentence structure of the original text. I do this latter only if I am quoting *or if the phrasing is standard technical phrasing used in the discipline* (as in "singnificance testing", "confidence intervals", "performing a *t*-test at the 5% confidence level", etc.)

3. A critique is very different to a summary or a paraphrase. In a critique we discern, or judge, from an academic point of view. We compare and contrast
    a. similarities and differences in the work done;
    b. significant and/or insignificant aspects of the work done;
    c. advantages or disadvantages of the work done;
    d. strengths and weakness, pros and cons of the work done;
    e. aspects of the work which are important or not, and why they are important or not;
    f. the usefulness of any solutions proposed. Here we evaluate whether or not such solutions are "good" or "bad", appropriate or inappropriate.

etc. Critiques are critical reflections, analyses or justifications of our own work and the work of others, and lead to ideas for future work, suggestions for improvement.

The above categories a. to d. form a type of comparative analysis of the text. In order to carry out such a comparative analysis we must first summarise the text. So, in general a critique can't be done without first doing a summary. As a result of performing one, or more, of a. to d. above critiques can be quite long.

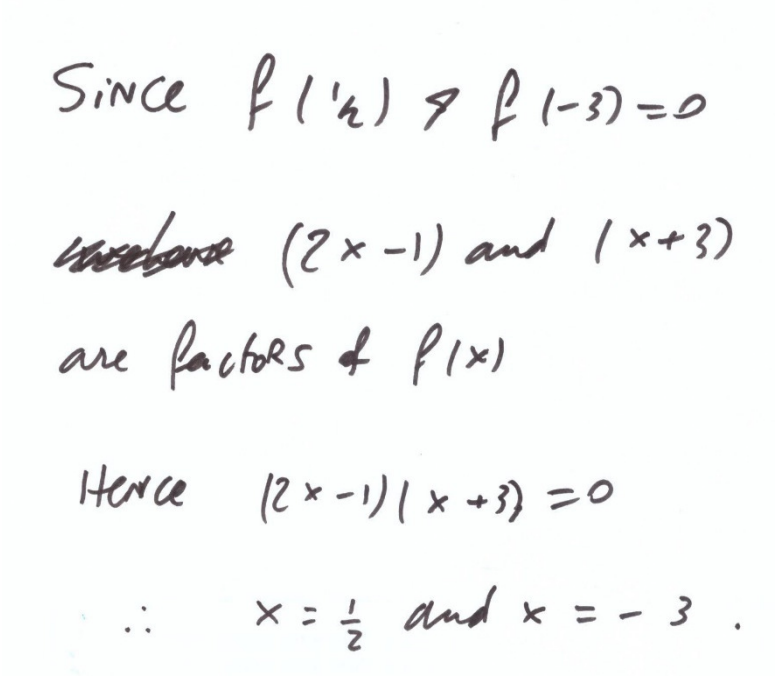
Other examples of critiquing the technical aspects of mathematics (optional)

Example 1

Consider the (hopefully) simple example of solving the following question:

Find the values of x which satisfy $2x^2 + 5x - 3 = 0$.

Solution



Critique of the solution

The first line shows the evaluation of the function at two different x value. Part of the first line is not written as an equation, and this is therefore not a correct presentation of maths.

- *First line of the solution*: In mathematics we need to justify what methods we are using before we use them. This has not been done for the first line of the solution. Here a method has been used without stating what the method is. Hence, we will need to specify that we are using the factor theorem.

  The other problem with the first line is that we should write separate equations for $f(½)$ and $f(–3)$ rather than have a "free-floating" $f(½)$ without an equals sign. So we should write "$f(½) = 0$ and $f(–3) = 0$"

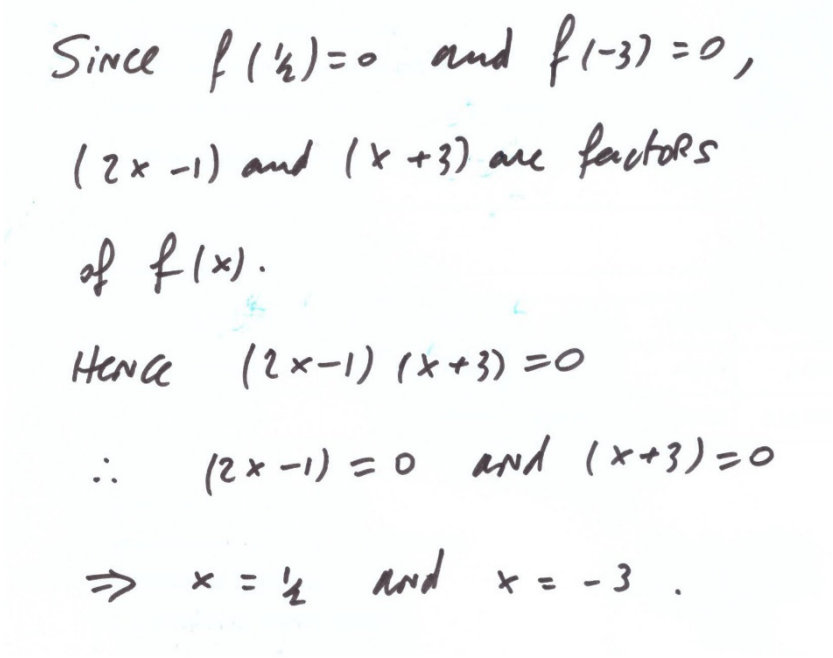
- *Second line of the solution*: There is a scratch mark at this step. There should be no scratch marks in the presentation of the solution.

- *Fifth line of the solution*: There is a step missing between the fourth and the fifth line. The question is, How do we get from the factored form of the fourth line to the actual

answers of the fifth line? In mathematics we need to explicitly show an intermediate step, this being to consider each fact equal to zero. Hence we should write "$(2x - 1) = 0$ and $(x + 3) = 0$" between the fourth and the fifth line.

Correct solution

The solution below address all of the issues discussed in the critique above.



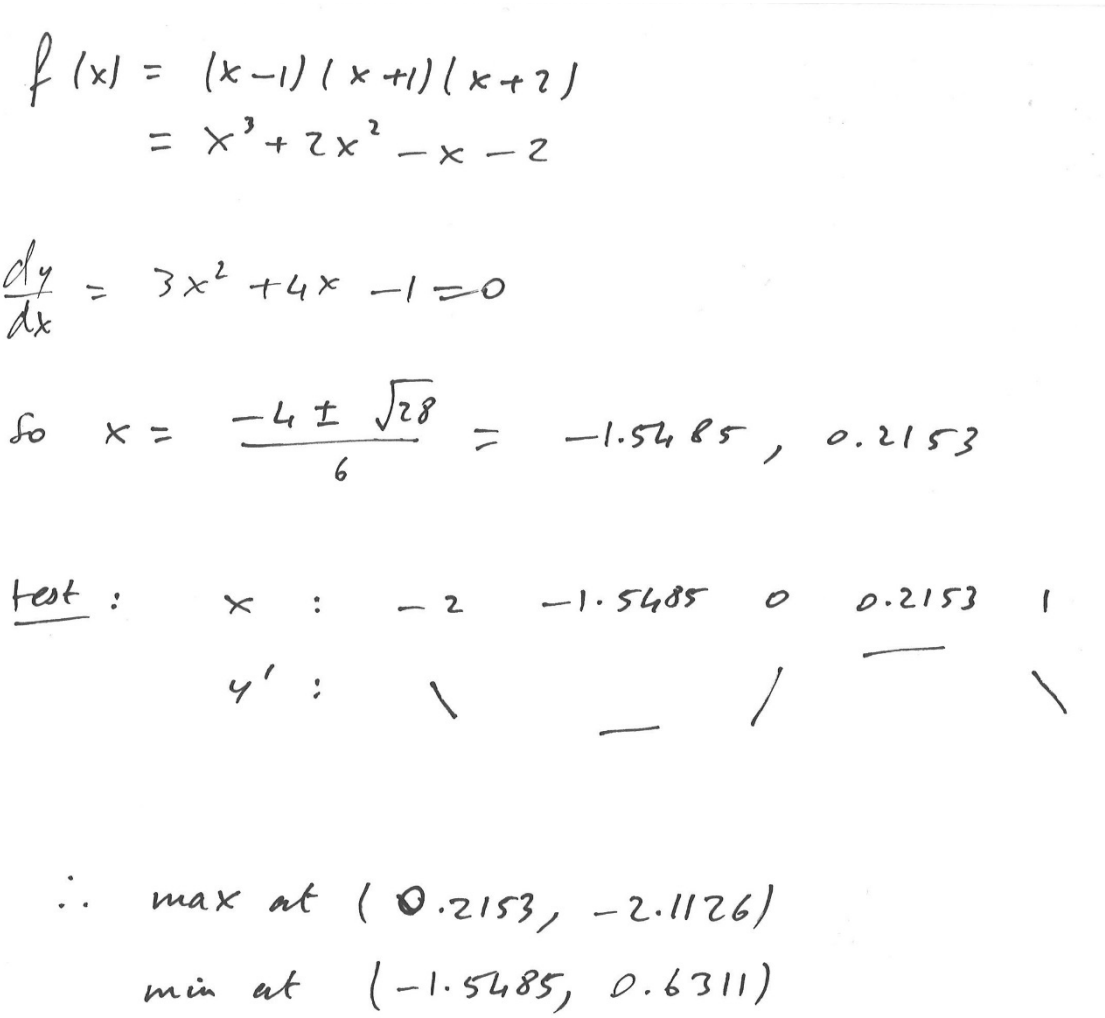
Exercise

Whatever your level of mathematical ability it should still be possible for you to recognise my critique as a critique (and not as a summary or paraphrase). What is it about my critique that makes it a critique? What type of language or phrasing or intention am I using in my critique? Did I use any of the forms of analyses a. to d. previously described? If I haven't used any of these forms of analysis what form of analysis do you believe I have used?

Example 2 (optional): More difficult?

Find and classify all the turning points of the function $f(x) = (x - 1)(x + 1)(x + 2)$.

Solution



Summary and critique: Exercise

The maths example above may be more difficult but my previous comment about being able to identify critique style still applies. So, can you identify in the commentary below all places where I am summarising and where I am critiquing?

> In this solution we start by expanding the factored form of $f(x)$. The next step then finds the first derivative of this function. However, there is an inconsistent use of notation since the symbol 'y' has been used instead of $f(x)$. In terms of mathematical presentation it is important to be consistent in one's use of notation. Therefore, it would be more appropriate to continue with the same

notation for the function as has been originally stated in the question, and write df/dx.

Also, the derivative is equated to zero at the same step as the derivative has been found. In general, when solving optimisation problems, these two aspects are presented as two different steps. In other words it is more appropriate to first write

$$\frac{df}{dx} = 3x^2 + 4x - 1$$

and then write

$$\text{for stationary points, } 3x^2 + 4x - 1 = 0$$

The next step involves simply solving this quadratic, from which we get the x ordinate of the stationary points.

After this comes the usual test for classifying stationary points. In this case the first derivative test has been used, but equally the second derivative test could have been used. Both tests have their advantages and disadvantages, namely that the second derivative test can sometimes be quicker, but may fail to give a valid answer in certain cases. The first derivative test will always work, but can take longer to apply, particularly if there are many turning point to have to test.

The last two steps of the solution classify the stationary points as well as giving their coordinates.

There is, however, a missing part to the solution. The question asked for all turning points of the function, and this solution has only presented three of the turning points (specifically the three that are called stationary points). It does not present the solution to the other two turning points which exists on this function, namely the point of inflection which exists between the maximum point and the minimum point. This is a serious omission in the presentation of a mathematical solution, and would need to be corrected before it could be considered a complete solution to the problem.

Note that you can write a summary or a paraphrase without doing a critique, but you can't write a critique without first summarising or paraphrasing certain aspects of the text. Why?

Critiquing the technicality of engineering, computer science, chemistry, etc.

I cannot present any example on critiquing these subjects since they are not my area of expertise. However, we can still take a first pass at listing some of the technical aspects for critique that are specific to these disciplines?

In-class exercise (if time allows)

Let us list some of the technical aspects which we could critique that are specific to the following disciplines. Choose the discipline relevant to you. If the discipline is not shown, then add it.

Mechanical engineering

1. = = =
2. = = =

Software engineering

1. = = =
2. = = =

Big data / Data analytics / Information security

1. = = =
2. = = =

Computer science

1. = = =
2. = = =

An example of critiquing a subject you know nothing about

The previous section brings us onto the issue of the degree to which it is possible to critique a subject we know nothing about. I contend that it is possible, to some degree, but obviously not to the depth of technicality that we could do so if we knew the topic.

To see how this might be, consider the following abstract taken from *An approach for the evaluation of energy and cost efficiency of glass facades*, Ikbal Cetiner, Ertan Ozkan, *Energy and Buildings 37 (2005) 673–684.*

> **Abstract**
>
> Glass facades, particularly in high-rise buildings, increase in energy consumption for heating, cooling and ventilation. This causes too high running cost of mechanical systems. Double skin glass facade is a system that decreases these disadvantages, by providing natural ventilation, preventing solar heat gain, controlling daylight, etc. This paper aims to investigate the appropriateness of double skin glass facades in moderate climate, such as Istanbul, in terms of the energy and cost efficiency when compared to single skin glass facades. For this purpose, an approach is proposed to determine the efficient alternatives. It comprises to generate standard facade alternatives by considering the objectives, constraints and performance criteria, and to evaluate their energy and cost efficiency for both single and double skin glass facades. In conclusion, the most energy efficient double skin glass facade is about 22.84% more efficient than the most energy efficient single skin glass facade is. Additionally, the most cost-efficient single skin glass facade is about 24.68% more efficient than the most cost efficient double skin glass facade is.

I can't provide a critique for this subject since I know nothing about the use of glass in buildings, the energy efficiency of glass, or life cycle costs in this context. But I can make an educated guess as to what I would consider critiquing, as follows:

- "Glass facades, particularly in high-rise buildings, increase in energy consumption for heating, cooling and ventilation. This causes too high running cost of mechanical systems. Double skin glass facade is a system that decreases these disadvantages, …"

Here I could question the expense of creating double skin facades over single skin facades. It seems logical the double skin facades would be more expensive to manufacture, but is this cost less than the total running costs in buildings having single skin facades?

- "Double skin glass facade is a system that decreases these disadvantages, by providing natural ventilation, preventing solar heat gain, controlling daylight, etc"

  Here I could analyse the way in which they measured natural ventilation, reduction in solar heat gain, and the control of daylight in order to find any better ways in which this might be done.

- "This paper aims to investigate the appropriateness of …"

  How do they measure or investigate "appropriateness"? Is their investigation appropriate? This is something I could look at.

- "For this purpose, an approach is proposed to determine the efficient alternatives. It comprises to generate standard facade alternatives by considering the objectives, constraints and performance criteria …"

  Are objectives, constraints and performance criteria the relevant aspects to consider in the context of single/double glass facades? If not why not? And how could things be done better?

The following is where most of your critiquing would be directed:

- "… the most energy efficient double skin glass facade is about 22.84% more efficient than the most energy efficient single skin glass facade is. Additionally, the most cost efficient single skin glass facade is about 24.68% more efficient than the most cost efficient double skin glass facade is."

The authors states percentage gains/improvements and energy and cost efficiencies. Here you would have to analyse and understand how they came to these numbers.

In other words, you would need to repeat their analysis according to your expertise to see if they have made any mistakes, assumptions, simplifications, omissions, (if any) etc. in their analysis, and then make recommendations as to how they could improve their work.

Exercise: Using any paper(s) of your choice identify the aspects for critique that are specific to your discipline. Compare these with your classmate's version. Are there any similarities and differences between your list and their list?

Comparing the aim of a critique to the aim of summaries or paraphrases

Refer back to any work you have already done on critique/critical thinking. Remind yourself of the way of thinking about critical thinking and critiquing, as well as the discourse of this. Compare this with the way of thinking and writing about a summary and a paraphrase. Summaries and paraphrases can be considered as descriptive writing, whereas a critique can be considered more as critical reflection, analysis or justification.

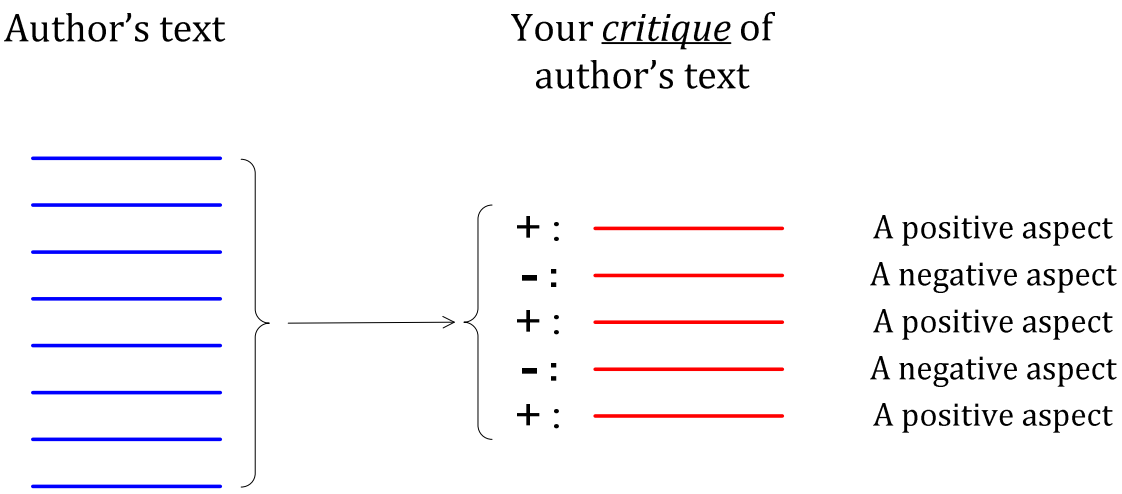
So, when writing a summary ask yourselves, Am I writing in a summary style? Is this the language/discourse of a summary? Ditto for a paraphrase and a critique: Am I writing in a paraphrase or critique style? Is this the language/discourse of a paraphrase or critique? A summary of the difference between descriptive writing (such as summaries and paraphrases) and a critique is shown in the table below.

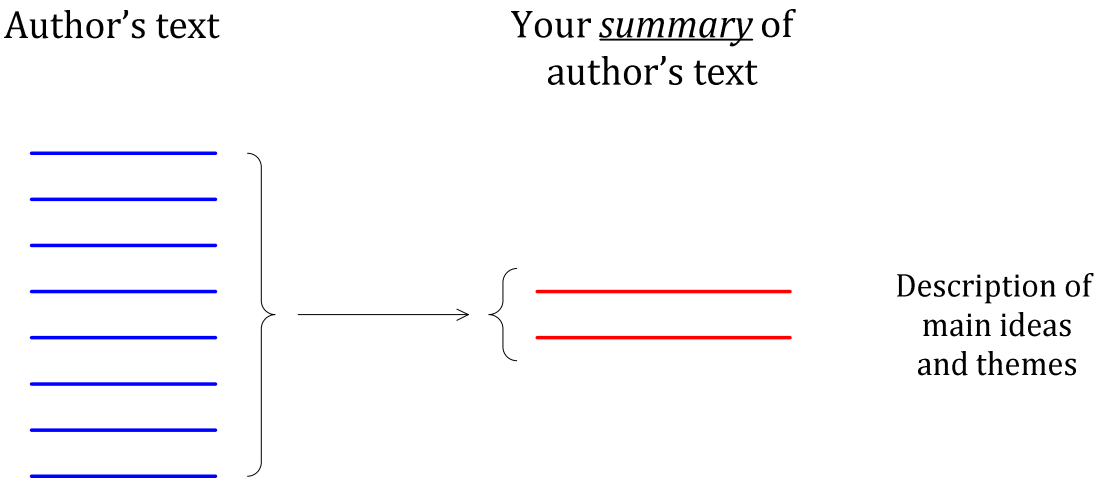
Descriptive writing	**Critical analytical writing**
States what happened	Identifies the significance of something that happened
States what something is like	Evaluates its strengths and weaknesses
States the order in which things happened	Structures information in order of importance
Explains what a theory says	Discusses the importance/failings/relevance of a theory in relation to a topic/idea
Explains how something works	Indicates why something will work (best)
Notes the methods used	Evaluates whether the extent to which the methods used were fit for purpose
Says when something occurred	Identifies why timing is of importance
States the different components	Weighs up the importance of component parts
States links between items	Shows the relevance of links between pieces of information
Gives information	Draws conclusions

Adapted from: Cottrell, S., (2008) *The Study Skills Handbook*, Hampshire: Palgrave Macmillan, p286.

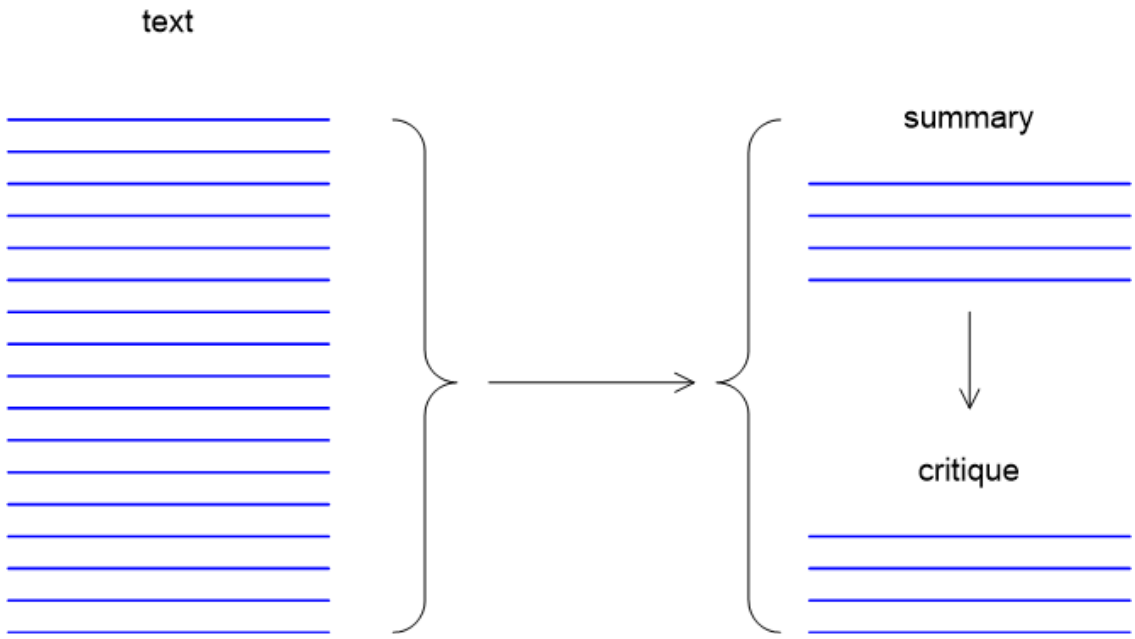
Possible ways of organising critiques and summaries

Below are diagrams which I have invented in order to illustrate possible ways of structuring one's summaries and critiques in an essay.





1) Summarise the text then critique the text.



2) Summarise and critique one part of the text at a time.